Supersymmetry and Vacuum Energy in Five-Dimensional Brane Worlds

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Abstract

We present the explicit form of the four-dimensional effective supergravity action which describes low-energy physics of the Randall–Sundrum model with moduli fields in the bulk and charged chiral matter living on the branes. The low-energy action is derived from the compactification of a locally supersymmetric model in five dimension. We describe the mechanism of supersymmetry breaking mediation which relies on the non-trivial configuration of the Z_2-odd bulk fields. Broken supersymmetry leads to stabilization of the interbrane distance.

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We present the explicit form of the four-dimensional effective supergravity action, which describes low-energy physics of the Randall–Sundrum model with moduli fields in the bulk and charged chiral matter living on the branes. The low-energy action is derived from the compactification of a locally supersymmetric model in five dimensions. We describe the mechanism of supersymmetry breaking mediation which relies on the non-trivial configuration of the Z_2-odd bulk fields. Broken supersymmetry leads to stabilization of the interbrane distance.

1. Introduction

This talk is based on results obtained in collaboration with Adam Falkowski and Stefan Pokorski, published in refs. [1–3].

Brane worlds with warped geometries offer new perspectives in understanding the hierarchy of mass scales in field theory models [4–6]. The initial hope was that the mere presence of extra dimensions would be a natural tool to control mass scales in gauge theories coupled to gravity. However, the realization of these simple ideas in terms of consistent models eventually called for quite sophisticated constructions, such as the brane–bulk supersymmetry that we are going to discuss in this talk.

The basic five-dimensional setup of brane world models is that of four-dimensional hypersurfaces (branes) hosting familiar gauge and charged matter fields, which are embedded in a five-dimensional ambient space, the bulk, populated by gravitational and gauge-neutral fields. The bulk degrees of freedom couple to the fields living on branes through various types of interactions. Some of these interactions are analogues of an in-

teraction between electromagnetic potential and charge density located on branes – this is the case of the fields that are Z_2-even on an S^1/Z_2 orbifold forming the fifth dimension; some of them are rather analogues of the derivative coupling of the potential to the electric dipole moment density located on branes, i.e. analogues of the interactions of the Z_2-odd fields on S^1/Z_2. These interactions lead to the formation of nontrivial vacuum configurations in the brane system. In particular, the solutions of Einstein equations of the form $ds^2 = a^2(x^5)ds_4^2 + b^2(x^5)(dx^5)^2$ are usually allowed, as in the original Randall–Sundrum (RS) models, where ds_4^2 is the Minkowski, anti-deSitter or deSitter metric in 4d. The two basic observations pertaining to the hierarchy problem are the following. First, on the brane located at a position x^5, all the fundamental mass scales defining the 5d Lagrangian become down-scaled by the factor $a(x^5)$: $m \to ma(x^5)$ when written down in the frame canonical with respect to the 4d line element ds_4^2. Thus, if the warp factor falls down exponentially, as in the RS model, one is given a natural exponential mass hierarchy between branes which is directly related to their separation. In fact, the effective mass measuring the interaction of a test body with the gravitational zero-mode is modulated by the warp factor, $m_{eff} = ma(x^5)$. In addition, the heavy Kaluza-

Klein modes of the metric tensor couple to the brane matter energy momentum tensor at x^5 with the strength $\Lambda^{-1}(x^5)$ where $\Lambda(x^5) = M_P a(x^5)$, thus implying, at least naively, an UV cut-off of that scale on perturbative physics on the brane. Second, as pointed out long ago by Rubakov and Shaposhnikov [7], the gradient energy associated with the variation of the warp factor in the direction transverse to the brane can cancel the contribution of the brane physics to the effective 4d cosmological constant. However, the stumbling observation is that whenever one finds a flat 4d foliation as the solution of higher-dimensional Einstein equations, which seems to be necessary for the existence of a realistic 4d effective theory, it is accompanied by a special choice of various parameters in the higher dimensional Lagrangian (see [8]). The fine-tuning seems to be even worse in 5d than in 4d, since typically one must correlate parameters living on spatially separated branes. Then there appears immediately the problem of stabilizing these special relations against quantum corrections. This situation has prompted the proposal [9,10,1], that it is a version of brane-bulk supersymmetry that may be able to explain apparent fine-tunings and stabilize hierarchies against quantum corrections. Indeed, the brane-bulk supersymmetry turns out to correlate in the right way the brane tensions and bulk cosmological constant in the supersymmetric Randall–Sundrum model. Moreover, local supersymmetry is likely to be necessary to embed brane worlds in string theory. Hence, the quest for consistent supersymmetric versions of brane worlds goes on, see [9,10,1,11,2,13–19,21–25,3,26]. First explicit supersymmetric models with delta-type (thin) branes were constructed in [9,1,2,13]. The distinguishing feature of the pure supergravity Lagrangians proposed in [1] is imposing the Z_2 symmetry, such that gravitino masses are Z_2-odd. An elegant formulation of the model is given in ref. [13] where an additional non propagating fields are introduced to independently supersymmetrize the branes and the bulk. In the on-shell picture for these fields the models of ref. [13] and refs. [1,2] are the same. On the other hand, in refs. [1,2] it has been noted, that supersymmetric Randall–Sundrum-type models can be gener-

alized to include the universal hypermultiplet and gauge fields and matter on the branes. The Lagrangian of such a construction has been given in [1–3,12]. This has allowed us [1,3] to study issues such as supersymmetry breaking and its transmission through the bulk. Finally, in [3] the effective low energy theory was formulated, which describes properly the physics of the warped five dimensional models with gauge sectors on the branes.

We have shown that in the class of models without nontrivial gauge sectors in the bulk, unbroken $N = 1$ local supersymmetry (classical solutions with four unbroken supercharges) implies vanishing of the effective cosmological constant. We have demonstrated the link between vanishing of the 4d cosmological constant, minimization of effective potentials in 5d and 4d, and moduli stabilization. We have also described supersymmetry breaking due to a global obstruction against the extension of bulk Killing spinors to the branes, which is a phenomenon observed earlier in the Horava–Witten model in 11d and 5d.

First steps towards the 4d effective theory were made in [24],[25] (where the Kähler function for the radion field was identified). In the set-up we consider in this paper supersymmetry in 5d is first broken from eight down to four supercharges by the BPS vacuum wall, and then again broken spontanously down to $N = 0$ due to switching on expectation values of sources living on the branes. The general strategy follows the one [27–30] that led to the complete and accurate description of the low-energy supersymmetry breakdown in the Horava–Witten models, see [27,30,31]. We were able to deduce the Kähler potential, superpotential and gauge kinetic functions describing physics of corresponding vacua in four dimensions. It turns out that the warped background modifies in an interesting way the kinetic terms for matter fields and the gauge kinetic function on the warped wall. There also appears a potential for the radion superfield, its origin being a modulus-dependent prefactor multiplying the superpotential on the warped wall in the expression for the 4d effective superpotential.

2. Supersymmetry in the brane-world scenarios

Let us begin with a brief review of the original RS model. The action is that of 5d gravity on $M_4 \times S_1/Z_2$, with negative cosmological constant:

$$S = M^3 \int d^5x \sqrt{-g}(\tfrac{1}{2}R + 6k^2)$$
$$+ \int d^5x \sqrt{-g_i}(-\lambda_1 \delta(x^5) - \lambda_2 \delta(x^5 - \pi\rho)). \quad (1)$$

Three-branes of non-zero tension are located at Z_2 fixed points. The ansatz for vacuum solution preserving 4d Poincare invariance has the warped product form:

$$ds^2 = a^2(x^5)\eta_{\mu\nu}dx^\mu dx^\nu + R_0^2(dx^5)^2. \quad (2)$$

The breathing mode of the fifth dimension is parametrized by R_0. The solution for the warp factor $a(x^5)$ is:

$$a(x^5) = \exp(-R_0 k|x^5|). \quad (3)$$

It has an exponential form, which can generate large hierarchy of scales between the branes. Matching delta functions in the equations of motion requires fine-tuning of the brane tensions:

$$\lambda_1 = -\lambda_2 = 6k. \quad (4)$$

With the choice (4) the matching conditions are satisfied for arbitrary R_0, so the fifth dimension is not stabilized in the original RS model. Thus R_0 enters the 4d effective theory as a massless scalar (radion), which couples to gravity in the manner of a Brans–Dicke scalar. This is at odds with the precision tests of general relativity, so any realistic model should contain a potential for the radion field.

Relaxing the condition (4) we are still able to find a solution in the maximally symmetric form, but only if we allow for non-zero 4d curvature (adS_4 or dS_4) [32]. In such a case the radion is stabilized and its vacuum expectation value is determined by the brane tensions and the bulk cosmological constant.

The Randall–Sundrum model can be extended to a locally supersymmetric model [9,10,1]. The basic set-up consists of 5d $N = 2$ gauged supergravity [33,34], which includes the gravity multiplet $(e^m_\alpha, \psi^A_\alpha, \mathcal{A}_\alpha)$, that is the metric (vielbein), a pair of symplectic Majorana gravitinos, and a vector field called the graviphoton. The 5d SUGRA action is

$$S = \int d^5x e_5 \tfrac{1}{\kappa^2} (\tfrac{1}{2}R - \tfrac{1}{2}\overline{\psi_\alpha}^A \gamma^{\alpha\beta\gamma}D_\beta\psi_{A\gamma}$$
$$- \tfrac{3}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + ...) \quad (5)$$

and the supersymmetry transformations are given by

$$\delta e^m_\alpha = \tfrac{1}{2}\overline{\epsilon}^A \gamma^m \psi_{A\alpha}$$
$$\delta\psi^A_\alpha = D_\alpha\epsilon^A - \tfrac{i}{4\sqrt{2}}(\gamma_\alpha^{\beta\gamma} - 4\delta_\alpha^\beta\gamma^\gamma)\mathcal{F}_{\beta\gamma}\epsilon^A$$
$$\delta\mathcal{A}_\alpha = -\tfrac{i}{2\sqrt{2}}\overline{\psi_\alpha}^A \epsilon_A.$$

Let us now add the brane tension at the brane located at $x^5 = 0$, $S_1 = \int d^5x e_4(-6k)\delta(x^5)$, and perform the supersymmetry transformation on the determinant of the induced vierbein. This produces a delta-type variation in the action: $\delta e \Rightarrow 3\delta(x^5)e_4 k(\bar{\psi}^1_\mu\gamma^\mu\epsilon^1 + (1 \leftrightarrow 2))$. It is straightforward to notice that this can be cancelled through the variation of the term $\bar{\psi}^A_\mu\gamma^{\mu5\rho}D_5\psi^A_\rho$ upon introducing new terms in the transformations of gravitini: $\delta\psi^1_\alpha = +\tfrac{k}{2}\epsilon(x^5)\gamma_\alpha\epsilon^1$, $\delta\psi^2_\alpha = -\tfrac{k}{2}\epsilon(x^5)\gamma_\alpha\epsilon^2$. These corrections introduce further variations in the bulk Lagrangian, which require further new terms in the bulk Lagrangian: $\mathcal{L}_{\psi^2} = +\tfrac{3e_5}{4}k\epsilon(x^5)(\bar{\psi}^1_\alpha\gamma^{\alpha\beta}\psi^1_\beta - \bar{\psi}^2_\alpha\gamma^{\alpha\beta}\psi^2_\beta)$ and $\mathcal{L}_{cc} = 6e_5k^2$, which is precisely the bulk potential needed in the RS model. The continuation through $x^5 = \pi\rho$ gives on the second brane the tension term $+\delta(x^5 - \pi\rho)e_46k$. The resulting locally supersymmetric Lagrangian is in fact that of a gauged supergravity. The symmetry that is gauged is the $U(1)$ subgroup of the R-symmetry, $\psi^A_\alpha \rightarrow e^{i\phi}\psi^A_\alpha$, the gauge field being $\mathcal{A}^R_\alpha = -\tfrac{1}{2\sqrt{2}}\mathcal{A}_\alpha$. Gauging of the $U(1)_R$ symmetry means that we replace the derivative acting on the gravitino with the $U(1)_R$ covariant derivative:

$$D_\alpha\psi^A_\beta \rightarrow D_\alpha\psi^A_\beta - \tfrac{3}{\sqrt{2}}(\sigma^3)^A_B k\epsilon(x^5)\mathcal{A}_\alpha\psi^B_\beta,$$

where D_α denotes the ordinary space-time covariant derivative. The coefficient of the coupling $\mathcal{A}_\alpha\psi^B_\beta$ defines the prepotential $\mathcal{P} = \tfrac{i}{4}\sigma^3$ and the Z_2-odd gauge coupling $g = \tfrac{6k\epsilon(x^5)}{\sqrt{2}}$. The warp factor turns out to be $a(x^5) = e^{-kR_0|x^5|}$, precisely

the one of the original RS model. Thus the fine-tuning present in the original RS model can be explained by the requirement of local supersymmetry [1].

New bosonic and fermionic fields do not affect the vacuum solution, so that the equations of motion for the warp factor are the same as in the original, non-supersymmetric RS model. The RS solution satisfies the BPS conditions and preserves one half of the supercharges, which corresponds to unbroken $N = 1$ supersymmetry in four dimensions. In the supersymmetric version the brane tensions are fixed. In consequence, the exponential solution (3) is the only maximally symmetric solution and the radion is still not stabilized.

We now turn to studying the supersymmetric RS model coupled to matter fields. We want to investigate how general the features present in the minimal supersymmetric RS model are. We find that unbroken local supersymmetry implies flat 4d space-time in a wider class of 5d supergravities coupled to hyper- or vector multiplets, in which the scalar potential is generated by gauging a subgroup of R-symmetry.

Let us add neutral matter in the bulk (moduli) in the form of a universal hypermultiplet, $(\lambda^a, V, \sigma, \xi, \bar{\xi})$. The quaternionic metric h_{uv} of the scalar manifold can be read from the Kähler potential: $K = -\ln(S + \bar{S} - 2\xi\bar{\xi})$, $S = V + \xi\bar{\xi} + i\sigma$. New kinetic terms in the bulk Lagrangian contain

$$S = \int d^5 x e_5 \frac{1}{\kappa^2} \left(-\frac{1}{4V^2}(\partial_\alpha V \partial^\alpha V + \partial_\alpha \sigma \partial^\alpha \sigma) \right.$$
$$\left. -\frac{1}{V}\partial_\alpha \xi \partial^\alpha \bar{\xi} - \frac{1}{2}\overline{\lambda}^a \gamma^\alpha D_\alpha \lambda_a + \dots \right). \quad (6)$$

Under Z_2, the bosonic fields (V, σ) are even, and (ξ) is odd. To arrive at supersymmetric Lagrangian in the presence of these additional fields it is necessary to gauge, in addition to $U(1)_R$, the isometry of the hypermultiplet scalar manifold: $\xi \to e^{i\theta}\xi$. Components of the Killing vector corresponding to this isometry are $k^\xi = i\xi$, $k^{\bar{\xi}} = -i\bar{\xi}$, $k^x = -y$, $k^y = x$, where $\xi = x + iy$. This results in the prepotential

$$\mathcal{P} = (\frac{1}{4} - \frac{x^2+y^2}{4V})i\sigma^3 - \frac{x}{2V^{1/2}}i\sigma^1 + \frac{y}{2V^{1/2}}i\sigma^2 \quad (7)$$

and scalar potential:

$$V = -k^2 \left(6 + \frac{3}{V}|\xi|^2 - \frac{3}{V^2}|\xi|^4 \right). \quad (8)$$

Finally, supersymmetry requires new terms localized on the branes: $\mathcal{L}_B = -\frac{e_4}{\kappa^2}6k(1 - \frac{|\xi|^2}{V})(\delta(x^5) - \delta(x^5 - \pi\rho))$ and modifications of supersymmetry transformation laws that were given in [3]. To discuss both unbroken and broken $N = 1$ supersymmetry, let us use from now on a more general ansatz for the metric allowing adS_4 geometry in four dimensions (the case of dS_4 is similar): $g_{\alpha\beta} = diag\,(-a^2(x^5)e^{2Lx^3}, a^2(x^5)e^{2Lx^3}, a^2(x^5)e^{2Lx^3}, a^2(x^5), R_0^2)$, where R_0 is the radion, and for later convenience we define $f = \exp(Lx^3)$. Let us note that $R^{(4)} = -12L^2$ and $\Lambda_{cc}^{(4)} = \langle V_{eff}\rangle/M_P^2 = -6L^2$. The conditions for unbroken supersymmetry in the bulk (BPS conditions) $\delta\psi_\alpha^A = \delta\lambda^a = 0$ correlate scalars, the warp factor, Killing vector and prepotential:

$$\frac{a'}{a} = -4kR_0\sqrt{\vec{P^2}}, \; h_{uw}\partial_5 q^w = 6kR_0\partial_u\sqrt{\vec{P^2}}$$
$$h_{uv}\partial_5 q^u \partial_5 q^v = 9R_0^2 k^2 h_{uv}k^u k^v. \quad (9)$$

After substituting these relations into the expression for the 4d action and integrating over x^5 one obtains 4d effective action of the form

$$S_4 = M_{PL}^2 \int d^4x(\frac{1}{2}\bar{R} - V_{eff})$$
$$M_{PL}^2 V_{eff} = -M^3 a^4(0)(24k\sqrt{\vec{P^2}}(0) - \lambda_1)$$
$$+M^3 a^4(\pi\rho)(-24k\sqrt{\vec{P^2}}(\pi\rho) - \lambda_2). \quad (10)$$

Unbroken supersymmetry corresponds to $\sqrt{\vec{P^2}} = P_3$, which immediately implies $V_{eff} = \langle R^{(4)}\rangle = 0$. This is a general result, which holds for any hypermultiplet manifold, not only for the universal one, which implies that unbroken SUSY excludes solutions with non-zero 4d space-time curvature. Only Minkowski foliations correspond to unbroken $N = 1$ supersymmetry and the converse is also true in the class of models which are discussed in ref. [3]. A further consequence is that stabilization of the radion field is impossible without breaking of residual four supercharges. It should be noted that in 4d supergravities one can a'priori obtain an anti-deSitter solution and preserve supersymmetry at the same time. Hence, compactifications of the supersymmetric RS scenarios yield a special subclass of 4d supergravities.

3. Supersymmetry breaking mediated by Z_2-odd fields and radion stabilization

In this section we investigate an alternative mechanism of supersymmetry breaking, similar to that studied in M-theoretical scenarios [30]. It is triggered by brane sources coupled to the scalar fields in the bulk, which are odd with respect to the Z_2 parity. One way to see that supersymmetry is broken is to notice that the Killing spinor cannot be defined globally. The odd fields are the agents that transmit supersymmetry breaking between the hidden and visible branes. We present below a general description of our mechanism and then apply it to a specific model of 5d gauged supergravity with the universal hypermultiplet.

To see how to break the remaining supersymmetry (corresponding to $N = 1$ in 4d) down to nothing ($N = 0$ in 4d) one should inspect the parts of supersymmetry transformations of bulk fermions that contain the Z_2-odd fields ξ and their transverse derivatives:

$$\delta\psi_5^1 = -\frac{1}{\sqrt{V}}\partial_5\xi\epsilon^2 + 2k\epsilon(x^5)(-\frac{Re(\xi)}{2\sqrt{V}}(\sigma^1)^{1B}$$
$$+\frac{Im(\xi)}{2\sqrt{V}}(\sigma^2)^{1B})\gamma_5\epsilon_B,$$
$$\delta\lambda^1 = +\frac{i}{\sqrt{2V}}\gamma^5\partial_5\xi\epsilon^2 + 3k\epsilon(x^5)V_u^{A1}k^u\epsilon_A. \quad (11)$$

It is clear that vacuum expectation values of ξ and $\partial_5\xi$ are parameters of the supersymmetry breakdown and the field ξ is the agent of local mediation of supersymmetry breakdown. Another way to see that ξ, $\partial_5\xi \neq 0$ imply completely broken SUSY is to notice that this means that the components $\mathcal{P}^1$ and $\mathcal{P}^2$ of the prepotential are non-zero as well. The BPS conditions give:

$$\epsilon^2 = \frac{\sqrt{\vec{P}^2}\gamma_5\epsilon^1 - P_3\epsilon^1}{P_1 - iP_2}. \quad (12)$$

As an immediate consequence we find that if P_1 or $P_2 \neq 0$, then the Killing spinor is non-chiral, and is a mixture of the even and the odd components of the SUSY parameter ϵ. But only the even component survives the Z_2 projection at the orbifold fixed points, hence the Killing spinor cannot be defined globally and supersymmetry is broken because of the 'misalignment' between the bulk and the brane supersymmetries. In other words, the projection of the general bulk Killing spinor

onto the brane would contain insufficient degrees of freedom to generate the minimal supersymmetry on the brane. Explicitly, bulk BPS conditions require

$$\frac{a'}{a} = -4kR_0\sqrt{\vec{P}^2}, \; h_{uw}\partial_5 q^w = 6kR_0\partial_u\sqrt{\vec{P}^2}, (13)$$

but matching delta functions in the equations of motion implies

$$\frac{a'}{a}(0) = -4kR_0P_3(0), \; \frac{a'}{a}(\pi\rho) = -4kR_0P_3(\pi\rho)$$
$$h_{uw}\partial_5 q^w(0) = 6kR_0\partial_u P_3(0)$$
$$h_{uw}\partial_5 q^w(\pi\rho) = 6kR_0\partial_u P_3(\pi\rho). \quad (14)$$

All these conditions are automatically satisfied if $\sqrt{\vec{P}^2} = P_3$. Thus $P \sim \sigma^3$ is equivalent to the RS fine-tuning, unbroken supersymmetry and 4d Poincare invariance. However, if $\mathcal{P}_1$ or $\mathcal{P}_2$ are non-zero, there are four matching conditions, but only three free parameters; generically we are not able to satisfy the boundary conditions, and thus supersymmetry is broken.

To excite ξ, $\partial_5\xi$ one needs to couple them to the branes. Details are given in [3]. Forgetting for a while about coupling to gaugino condensates on branes, the relevant part of the brane–bulk coupling is

$$- \int d^5x \, e_5\frac{2}{Vg_{55}}(\delta(x^5)W_1(\partial_5\xi + 2\delta(x^5)\bar{W}_1)$$
$$+\delta(x^5 - \pi\rho)W_2(\partial_5\xi + 2\delta(x^5 - \pi\rho)\bar{W}_2) + h.c.).$$
$$(15)$$

We solve equations of motion perturbatively to order $(\frac{W}{M^3})^2$ with adS_4 foliation and ansatz $\xi = \xi(x^5) = \epsilon(x^5)\zeta(|x^5|)$, $V = V(x^5)$, $\sigma = const.$, $\mathcal{A}_\mu = 0$, $\mathcal{A}_5 = const.$ Matching the δ' in EOMs yields the boundary conditions: $\zeta(0) = -\frac{\bar{W}_1}{M^3}$, $\zeta(\pi\rho) = \frac{\bar{W}_2}{M^3}$. As a consequence:

$$C = -\frac{\bar{W}_1}{M^3}, \quad Ce^{k(R_0 - 3\sqrt{2}i\mathcal{A}_5)\pi\rho} = \frac{\bar{W}_2}{M^3}. \quad (16)$$

One can see that as long as supersymmetry is unbroken, moduli R_0 and $\mathcal{A}_5$ are arbitrary. When sources are switched on for odd fields, then the expectation value of the radion is determined by the boundary sources W_i, assuming that V gets frozen. Perturbative ($o(|W^2|)$) solution to EOMs

is

$$\xi = C\epsilon(x^5)e^{k(R_0-3\sqrt{2}ik\mathcal{A}_5)|y|}, \ C = -\frac{\overline{W}_1}{M^3},$$

$$Ce^{k(R_0-3\sqrt{2}ik\mathcal{A}_5)\pi\rho} = \frac{\overline{W}_2}{M^3}$$

$$a = e^{kR_0|y|} + \frac{|C|^2}{2V}e^{-kR_0|y|}, \ L^2 = \frac{16}{6}\frac{k^2|C|^2}{V}$$

$$V = V_0 - |C|^2e^{2kR_0|y|}, \ \sigma = \sigma_0, \qquad (17)$$

except that boundary conditions for V are fulfilled at the zeroth order only; hence it is better to say that one assumes V to be 'frozen'. The four-dimensional curvature is $R^{(4)} = -32\frac{k^2|C|^2}{V}$. This means that in our family of models broken supersymmetry implies negative 4d curvature, and that curvature vanishes only if the SUSY-breaking sources are switched off.

As a cross-check of the above results we can calculate the 4d effective potential, obtained by integrating out the 5d bosonic action in the background (17). The result (to the order $\frac{W}{M^3}^2$) is:

$$\mathcal{L}_4 = \sqrt{-\bar{g}}\frac{M^3}{k}(1-e^{-2kR_0\pi\rho})\left(\frac{1}{2}\bar{R} + 8\frac{k^2|C|^2}{V_0}\right).(18)$$

We denoted by $\bar{g}$ the oscillations of the 4d metric around the vacuum solution. Solving the Einstein equations in the 4d effective theory yields $\bar{R} = -32\frac{k^2|C|^2}{V_0}$, which is consistent with the value of $L^2 \equiv -\frac{1}{12}\bar{R}$ in (17). We also see that V_0 enters the denominator of the effective potential, which explains its runaway behaviour commented on earlier.

Before closing the discussion of the 5d classical solutions and supersymmetry breakdown, let us comment on proposals [19,20,35] to solve the cosmological constant problem by virtue of supersymmetry of the bulk-brane system. To put the issue into perspective, let us note that the Einstein equation with indices (55) does not contain second derivatives of fields, hence it acts as a sort of constraint on the solutions of the remaining equations. This becomes clearer in the Hamiltonian approach towards the flow along the fifth dimension, where this equation arises as the Hamiltonian constraint $\mathcal{H} = 0$, and is usually used to illustrate the way the conservation of the 4d curvature L^2 is achieved through the compensation between gradient and potential terms along the classical flow. However, this classical conservation hinges upon fulfilling certain consistency conditions between brane sources, or between boundary conditions induced by them, as illustrated by the model above. When one perturbs the boundary terms on one wall, then to stay within the family of maximally symmetric foliations one of two things must happen. Either the distance between branes must change, or the source at the distant brane must be retuned. In the class of models which we constructed, if the 4d curvature is present then supersymmetry is broken, and doesn't take care of such a retuning. Moreover, even if retuning takes place, the size of 4d curvature, i.e. of the effective cosmological constant, does change as well, moreover, the magnitude of the effective cosmological constant has quadratic dependence on the boundary terms which induce supersymmetry breakdown. Hence, any perturbation of the boundary, instead of being screened by the bulk physics, contributes quadratically to the effective cosmological constant. Of course, we are talking about perturbations which can be considered quasi-classical on the brane. Thus we do not see here any special new effect of the extra dimension in the the cancellation of the cosmological constant. The positive aspect of supersymmetry is exactly the one which we know from the 4d physics. Supersymmetry, even the broken one, limits the size of the brane terms inducing supersymmetry breakdown, and limits in this way the magnitude of the 4d cosmological constant, since the two effects are strictly related to each other.

4. Effective low-energy theory in four dimensions

In this section we give the form of the effective four-dimensional supergravity describing zero-mode fluctuations in the models presented in the previous sections. Since in the 5d set-up supersymmetry is broken spontaneously, it is safe to assume that in 4d this supersymmetry breakdown can be considered as a spontaneous breakdown in a 4d supergravity Lagrangian described with the help of a Kähler potential K, superpotential W and gauge kinetic functions H. The goal is to identify these functions reliably start-

ing from the maximally symmetric approximate solutions (17) that we have described in the previous section. Our procedure is perturbative in the supersymmetry breaking parameter $\frac{W}{M^3}$. Fortunately, it is sufficient to identify the functions we are looking for from the terms that can be reliably read at the order $(\frac{W_1}{M^3})^1$. Such terms include the gravitino mass term. In addition, we have at our disposal the complete kinetic terms for moduli, gauge and matter fields, which are of order $(W_1)^0$ and are sufficient to read off the Kähler potential for moduli and matter fields. The complete procedure has been given in ref. [3]. Here we summarize the results and discuss the basic features of the warped 4d supergravities.

The Kähler function for moduli fields $S = V_0 + i\sigma_0$ and $T = k\pi\rho(R_0 + i\sqrt{2}\mathcal{A}_5)$, and for the charged fields Φ_1 and Φ_2 living on the Planck brane and warped brane respectively is

$$K(S,\bar{S};T,\bar{T};\Phi,\bar{\Phi}) = -M_P^2\log(S+\bar{S})$$
$$-3M_P^2\log(f(T+\bar{T}-\tfrac{k}{3M^3}|\Phi_2|^2) - \tfrac{\beta k}{3M^3}|\Phi_1|^2) \quad (19)$$

where we defined $M_P^2 = \frac{M^3}{k}(1 - e^{-2k\pi\rho\langle R_0\rangle})$, $f = \beta(1 - e^{-(T+\bar{T})})$ with $\beta = \frac{M^3}{kM_P^2}$. The effective 4d superpotential is

$$W = 2\sqrt{2}(W_1 + e^{-3T}W_2) \quad (20)$$

and the gauge kinetic functions are

$$H_{warped}(S,T) = S + 2b_0 T, \quad H_{Planck}(S) = S. \quad (21)$$

One can study the 4d effective scalar potential derived from K, W, H. The standard formulation gives: $\mathcal{V} = e_4 e^G(G_i G^{i\bar{j}} G_{\bar{j}} - 3)$ with $G = K + \log|W|^2$. Explicitly:

$$\mathcal{V} = e_4 \frac{4}{V_0 M_P^2 \beta^3 (1 - e^{-2k\pi\rho R_0})^3}(|W_1|^2$$
$$(3e^{-2k\pi\rho R_0} - 2) + |W_2|^2(3e^{-4k\pi\rho R_0} - 2e^{-6k\pi\rho R_0})$$
$$+ W_1\bar{W}_2 e^{-3k\pi\rho R_0 - i\sqrt{2}\mathcal{A}_5}$$
$$+ W_2\bar{W}_1 e^{-3k\pi\rho R_0 + i\sqrt{2}\mathcal{A}_5}). \quad (22)$$

Minimizing the above scalar potential wrt $\mathcal{A}_5$ yields

$$Arg(W_2) - Arg(W_1) - 3\sqrt{2}k\pi\rho\mathcal{A}_5 = \pi n,$$
$$n = 0, \pm 1, \pm 2, \dots. \quad (23)$$

Taking $n = 1$ and minimizing wrt R_0 one obtains

$$e^{-k\pi\rho R_0} = \frac{|W_1|}{|W_2|}, \quad (24)$$

consistently with the 5d picture. Let us summarize the basic features of our model. The F-terms take at the minimum the expectation values

$$|F^S|^2 = 8e^K(S+\bar{S})^2 a^2(\pi\rho)|W_2|^2$$
$$\times(1 - a^2(\pi\rho))^2 \neq 0$$
$$|F^T|^2 = 0, \quad (25)$$

which means that supersymmetry is broken along the dilaton direction. The potential energy at this vacuum is negative:

$$V_{vac} = -\frac{8|W_2|^2}{V_0(M^3/kM_P^2)^3 M_P^2}\frac{a^2(\pi\rho)}{1 - a^2(\pi\rho)}. \quad (26)$$

The mass of the canonically normalized radion is

$$m_R^2 = \frac{24}{V_0(M^3/kM_P^2)^3}\frac{a^2(\pi\rho)|W_2|^2}{(1 - a^2(\pi\rho))}\frac{1}{M_P^4} \quad (27)$$

and the gravitino mass term is given by the expression

$$m_{3/2} = \frac{2}{\sqrt{V_0}(M^3/kM_P^2)^{3/2}}\frac{a(\pi\rho)}{(1 - a^2(\pi\rho))^{1/2}}\frac{|W_2|}{M_P^2}. \quad (28)$$

It is interesting to compare these features to those of the models, which are low-energy limits of weakly and strongly coupled heterotic string theories. As is well known, in the leading, tree-level, approximation, heterotic string gives the four-dimensional supergravity which enjoys the no-scale structure [36]. The gauge kinetic functions in all, visible and hidden, sectors, are universal and depend only on the 4d dilaton superfield S, $H = S$, $\partial_T H = 0$. In addition, at the perturbative level there is no superpotential for moduli superfields S and T. As a consequence, the effective potential is positive semi-definite and takes the form $V = K_{S\bar{S}}|F^S|^2 \geq 0$. Hence, the vacuum configuration corresponds to $F^S = 0$, but F^T, which doesn't enter the potential, is allowed to take a non-zero value, so that supersymmetry is broken along the T direction in moduli space. The problem with this is that the scale of supersymmetry breakdown is arbitrary, and it can only be hoped that it becomes fixed at a proper

value, after taking into account various perturbative and/or nonperturbative corrections to the Lagrangian. In the supergravity model which is the low energy limit of weakly coupled heterotic string with one-loop corrections, and at the same time that of a strongly coupled heterotic string where these corrections arise in classical expansion taking into account the presence of an extra dimension, see [37], the situation changes. When the interplay of sources of supersymmetry breakdown, such as condensates in various sectors and expectation value of the superpotential, is taken into account, it is possible to arrange for unbroken supersymmetry in the anti-deSitter background, or for vacua where both F^S and F^T are sizable - see [38,39] for details. In all these considerations the effects of the nonperturbative warping of an extra dimension, like the one in the RS models, were not taken into account. The model which was constructed in [3] finally allows to discuss the impact of the rapidly changing warp-factor on the low-energy physics. In the model with just expectation values of the superpotentials serving as a supersymmetry breaking source, it is crucial that the effective superpotential acquires the exponential dependence on the modulus T. This leads to vanishing of the F^T upon using the equation of motion for T. As a result, this time it is F^S which is non-zero, although its value remains undetermined (as long as the corrections to the Lagrangian are not taken into account). The vacuum energy takes the form $V = -\frac{2}{M_P^2} e^K |W|^2 \leq 0$, which describes, at tree-level, an unstable background with negative energy density, in agreement with five-dimensional considerations.

5. Summary

The main result of [3] summarized in this talk is the four-dimensional effective supergravity action, which describes low-energy physics of the Randall–Sundrum model with moduli fields in the bulk and charged chiral matter living on the branes.

The low-energy action has been read off from a compactification of a locally supersymmetric model in five dimensions. The exponential warp factor has interesting consequences for the form of the effective 4d supergravity. The asymmetry between the warped and unwarped walls is visible in the Kähler function, in the gauge kinetic functions and in the superpotential. Roughly speaking the contributions to these functions which come from the warped wall are suppressed by an exponential factor containing the radion superfield. This is the way the warp factor and (and RS brane tensions) is encoded in the low-energy Lagrangian.

We have described the mechanism of supersymmetry breaking mediation which relies on a nontrivial configuration of the Z_2 odd fields in the bulk. We point out, that the odd Z_2 parity fields can be an important ingredient of 5d supersymmetric models. They play a crucial role in communication between spatially separated branes.

After freezing the dilaton, it is possible to stabilize the radion field in the backgrounds with broken supersymmetry and excited odd-parity fields.

We have shown that in the class of brane world models without charged matter and gauge fields in the bulk, unbroken $N = 1$ supersymmetry implies vanishing cosmological constant. In the case where the sources that induce supersymmetry breakdown are represented simply by constant superpotentials on the branes, broken supersymmetry gives rise to anti-deSitter-type geometry in four dimensions. Hopefully, in more sophisticated models with various sources of supersymmetry breaking participating in the game, we shall be able to find models with broken supersymmetry and vanishing vacuum energy.

We believe that the class of models we have constructed in [3] provides a useful, explicit, setup to study low-energy phenomenology of the supersymmetric brane models with warped vacua.

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